RESEARCH OF HYDRODYNAMIC LOADINGS AT IMMERSING OF SECTIONALLY FLAT BOTTOM CONTOURS

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ABSTRACT

The present paper is devoted to the construction of nonlinear unsteady hydrodynamics mathematical model of bodies during interaction with free water surface. The using of vortex sheets method is supposed. Wetted surface of bottom and free fluid surface are represented by vortex sheets. The fluid perturbed motion velocity is described by means of integral in Cauchy form and singular-integral equation for bottom vortex density is constructed. The mathematical model of ship waves is constructed in order to determine the free surface vortex density. The summary and distributed loads as well as the laws of motions for the wedges of infinite and finite mass are obtained and compared with known experimental and theoretical results. Numerical experiment results on base of coming mathematical model may be taken as scientific basis for the prediction of slamming loads and designs of high-speed ships.

Keywords: *immersing of bottom, mathematical modeling of hydrodynamic loadings, vortex sheets method.*

1. INTRODUCTION

Many tasks of modern hydrodynamics are bound to driving of bodies in a fluid with free boundaries, the form which one is determined during solution and on which one the nonlinear boundary conditions should satisfy in a general case. The known research of such tasks, as a rule, are limited to the simplified kinematics and geometry of a body or linearized representation of free boundaries [1, 2, 3 etc.]. The problem of calculation of nonlinear hydrodynamic parameters of bodies of the complicated form intersecting a free surface of a weighty fluid is important for the majority of technical applications.

To the given paper the nonlinear non-stationary mathematical model and results of numerical research of imbedding of wedges of infinite width is described. The matching with known designed and experimental data is given.

2. FORMULATION OF PROBLEM

Let us assume that non viscous incompressible fluid is posed in a lower half-space with boundary which consists of a wetted part of a body $S_1(t)$, isobaric free surface $S_2(t)$ and hydrodynamic body wake $S_3(t) \subset S_2(t)$. Not upsetting a generality it is possible to suppose the known law of motion, including deformation, of a body and to consider the non-stationary flow with the given initial form of free boundary and wetted part of a body at a contact by it of a fluid.

A field of perturbation velocities in internal points of a fluid is potential on a Lagrange's theorem. A condition of impenetrability is satisfied on $S_1(t)$ and a dynamic condition of isobar is satisfied on $S_2(t)$.

As generally free boundary is not a stream-surface, in points of boundary the equation of the Euler should be fulfilled. The kinematical condition on free boundary is provided with its moving together with fluid particles. The fluid velocity is final in all points of $S_1(t)$ (generalization of the Chaplygin–Joukovsky postulate). Perturbation velocities of a fluid should equal to null at infinity outside of a hydrodynamic wake. According to the Thomson theorem we shall demand equality to null of a velocity circulation on an arbitrary fluid outline including area of disturbed motion.

3. MATHEMATICAL MODEL

Let us represent boundary of a fluid $S = S_1 \cup S_2, S_3 \subset S_2$ by a surface vortex pattern $\vec{\gamma}(S) = \{\vec{\gamma}_1(S_1), \vec{\gamma}_2(S_2), \vec{\gamma}_3(S_3)\}$.

When determining a velocity potential of disturbed motion in an upper half-space, we shall receive special analog of the Sohotsky formulas

$$\vec{w}^+(\vec{x},t) - \vec{w}^-(\vec{x},t) = \vec{\gamma}(\vec{x},t) \times \vec{n}(\vec{x},t);$$

$$\vec{w}^+(\vec{x},t) - \vec{w}^-(\vec{x},t) = \frac{1}{2\pi}\int_S \frac{\vec{\gamma}(\vec{\xi},t) \times (\vec{x}-\vec{\xi})}{|\vec{x}-\vec{\xi}|^3} dS,$$

where $\vec{w}^{\pm}(\vec{x},t)$ – limiting values of a fluid perturbation velocity at tendency of a point x to S accordingly from an upper (lower) half-space, $\vec{n}(\vec{x},t)$ – external in relation to a fluid a normal to S.

It is possible to determine the module $\vec{w}^-(\vec{x},t)$ of velocity on S_2 from the Cauchy–Lagrange integral, and its direction can by determined from the analysis of the

Euler equation due to orthogonality of a pressure gradient to isobaric free surface. In particular, a tangent component of fluid velocity on S_2 is collinear with projection of the acceleration due to gravity g, and their directions coincide for falling sides of surface and are opposite – for rising sides. This outcome is conformed to kinematics of Gerstner waves.

A impenetrability condition of a wetted part of a body and the Sohotsky formulas in view of continuity of normal velocity component on free surface S_2 allow to construct the system of the singular integro-differential equations for definition of vortex sheets density and forms of free surface, the wake and the body wetted surface.

$$\vec{n}(\vec{x},t)\int_S \frac{\vec{\gamma}(\vec{\xi},t)\times(\vec{x}-\vec{\xi})}{\left|\vec{x}-\vec{\xi}\right|^3}dS = 4\pi\vec{n}(\vec{x},t)\vec{v}_1(\vec{x},t), \qquad \vec{x}\in S_1(t);$$

$$\vec{\gamma}_2(\vec{x},t) = 2\vec{n}(\vec{x},t)\times(E-\vec{n}\vec{n}^{\mathrm{T}})\left\{\int_S \frac{\gamma(\vec{\xi},t)\times(\vec{x}-\vec{\xi})}{\left|\vec{x}-\vec{\xi}\right|^3}dS + \frac{\mathrm{sgn}(w_n(\vec{x},t))}{\left|\vec{n}(\vec{x},t)\times\vec{g}\right|}\vec{g}\left[v^2(t)-\left(w_n(\vec{x},t)-v_n(t)\right)^2 - 2\left(\dot{\varphi}(\vec{x},t)+gh(\vec{x},t)\right)\right]^{\frac{1}{2}} - \vec{v}(t)\right\}, \qquad \vec{x}\in S_2(t);$$

$$\frac{d\vec{x}}{dt} = \frac{1}{4\pi}\int_S \frac{\vec{\gamma}(\vec{\xi},t)\times(\vec{x}-\vec{\xi})}{\left|\vec{x}-\vec{\xi}\right|^3}dS - \frac{1}{2}\vec{\gamma}(\vec{x},t)\times\vec{n}(\vec{x},t) - \vec{v}(t), \qquad \vec{x}\in S_2(t),$$

where $\vec{v}(t)$ – velocity of an origin of body coordinates; $\vec{v}_1(\vec{x},t)$ – velocity of points of a body surface S_1; $h(\vec{x},t)$ – perturbation of free surface S_2; E – unit matrix; $(E-\vec{n}\vec{n}^{\mathrm{T}})$ – operator of projection of a vector on a surface, the multiplying on the transposed normal vector $\vec{n}^{\mathrm{T}}$ is understood in sense of multiplying of matrixes; the point marks a local derivative on time.

4. NUMERICAL IMPLEMENTATION OF MATHEMATICAL MODEL

At numerical implementation of a mathematical model the integrals are conversed to multivariate integrals such as the Cauchy [4], many properties which one are generalization of known outcomes of a one-dimensional case [5]. Let's suppose surfaces S_1 and S_2 piecewise smooth on Lyapunov, admitting only lines of angular points (location of a spraying on a water-line $S_1 \cap S_2$, keel both bilges on a body S_1 and nonlinear wave crests on S_2. Possible zones of so-x called out-of-limit nonlinearity (cusp and collapse points of wave crests, free jets etc.) thereby are eliminated from reviewing.

Postulating continuity of a velocity field shall demand vanishing of density of integrals such as Cauchy on lines of angular points, that corresponds condition on edge as Sommerfeld [6] for keel both bilges on a body S_1 and indirectly reflects collapse of nonlinear waves and free jets on S_2. On a part of a water-line $S_1 \cap S_2$ the appearance of spray jets is generally possible, the speed in which ones is finite [7]. The tops of these jets are angular points; it is possible and indefinitely remote. Density of an integral such as the Cauchy in them also should vanish [8]. We shall consider that vortex sheets density is satisfied the Gelder condition.

The numerical implementation of an offered mathematical model was realized by a method of discrete vortexes on algorithm of iterative approximation. At build-up of vortex model of a wetted surface of a body and wake the requirements of the Thomson and Helmholtz theorems were satisfied. The spray jets were simulated by additional vortexes in a water-line neighbourhood under the Joukovsky scheme [9] for obtaining solution with restricted load in a spray zone. The indispensable methodical researches of the calculating scheme of a wetted surface were conducted. At small trajectory corners of a body splashdown the rediscretization of a diving time step was envisioned.

It is known that the considered task with the initial data (Cauchy problem) for an unlimited time slice is ill-conditioned because of instability of free vortex surfaces [10]. Conditionally corrected numerical implementation of a mathematical model was obtained due to special regularization ways.

At first, usage of a mathematical formalism of the theory of integrals such as the Cauchy, allows considering zero vortex sheets density in free surface angular points and ensures its stability. Secondly, for long processes of interaction of a body with free surface, in particular, at diving, it turned out necessary to execute the Courant–Friedrichs–Levi condition [11] and in appropriate way to distort the calculating scheme.

The form and sizes of a body wetted surface can here noticeably vary as against the conventional tasks of hydrodynamics. That not only adds nonlinearity to a formulation, but also considerably complicates the solution of task. In particular, conformity of points at finite-difference calculation of a local derivative of potential can be ensured only at usage of an affine similarity with a current characteristic size of a wetted surface. Besides, at calculation of non-stationary loads on a body it is necessary to allow for rapidity of wetted surface change. In some cases it appears to expedient consideration of an integral of the Cauchy–Lagrange in a moving axis, bound with a characteristic point of a water-line.

5. CALCULATIONS

With the purpose of testing of a mathematical model

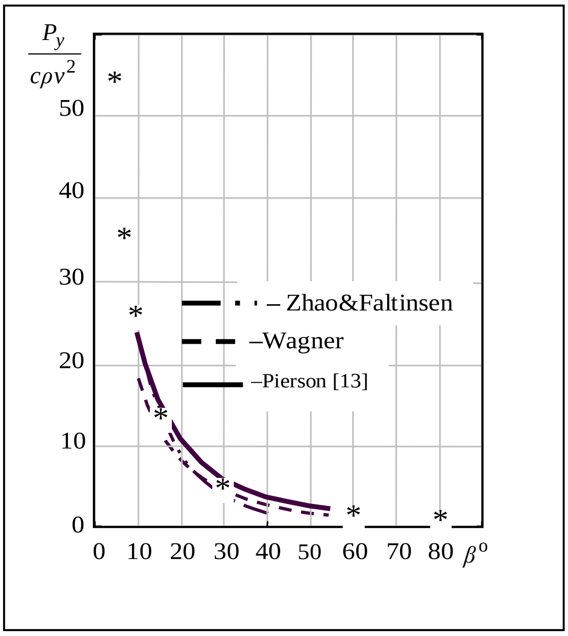

Figure 1 The calculation results of resistance force of wedges with different deadrise angles

the known non-stationary tasks about a start of gliding (Wagner's task) and vertical diving of a plate with constant velocity and trim angle were considered.
The calculation results of a gliding were compared to precise Sedov's solution for a weightless fluid. In calculation, the rarefaction in an after-body at the beginning of gliding is obtained, what was theoretically.
forecasted by Wagner and was watched by Sokolyansky and Malyarova in experiments of CAHI. The calculation results of a plate diving are satisfactorily agreed with experiment of Shorigin (CAHI). For forward edge of a plate a rarefaction area also is detected.

The task about a gliding of a deformable plate making elastic vibrations was considered. The time dependences of coefficients of normal force, moment, position of pressure center and pressure distribution on a plate for different parameters of oscillations is obtained. Vertical diving of symmetric wedges with different deadrise angles was considered. The calculation results of resistance force and pressure distribution at diving with constant velocity are shown on Figure 1 and Figure 2, in comparison to known theoretical estimations for a weightless fluid.

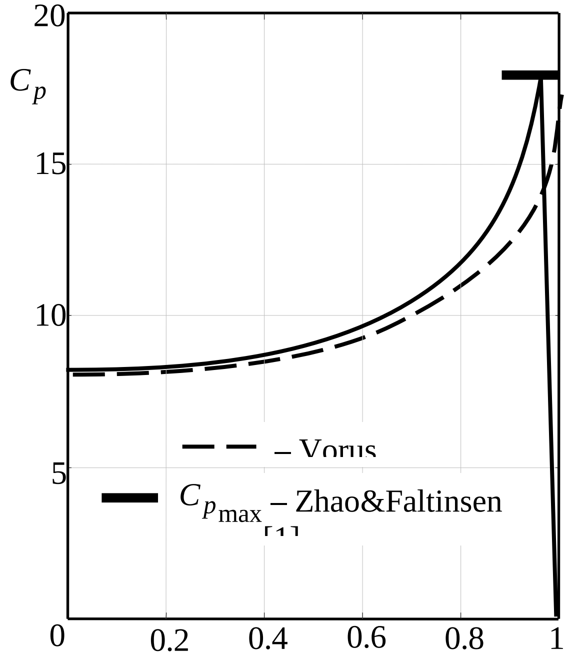

Figure 2 The calculation results of pressure distribution on a wedge with deadrise angles $\beta = 20^o$

The task about a diving wedge of a final mass was solved at final Froude numbers. The integral and distributed hydrodynamic loads, and also law of motion for different deadrise angles were calculated. The results of calculation of overload n_y and law $H(t)$ of diving for a wedge with a deadrise angle 30^o, linear mass density 112 kg/m and initial velocity 2.44 m/s in comparison to the experimental data of Shorigin [14] are shown in Figure 3. As in experiment the law $H(t)$ of

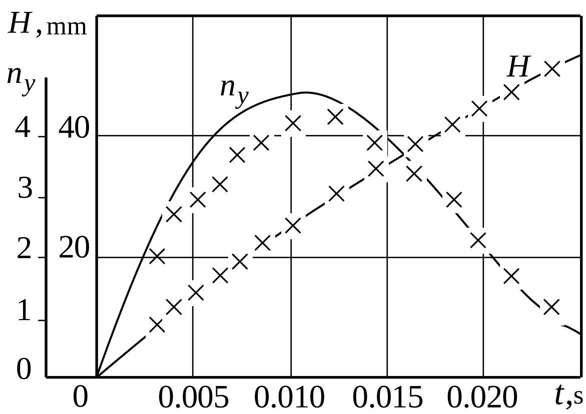

Figure 3 Diving of a wedge with finite mass: points – experiment of Shorigin [14]

diving (decryption of high-speed filming) was most authentically defined, the satisfactory coordination with it demonstrates that the calculated overload is defined more precisely than in physical experiment.

$\vec{v}$	Diving velocity
p_∞	Pressure on free boundary of a fluid
ρ	Mass density of a fluid
P_y	Resistance force of a wedge
H	Imbedding a wedge
G	Weight of a wedge
n_y	Overload $n_y = P_y / G$
c	Wetted half-width of a wedge
β	Deadrise angle of a wedge
C_p	Pressure coefficient $C_p = (p - p_\infty)/\rho v^2/2$

6. CONCLUSION

The built mathematical model can be applied for definition of loads with the purpose of an estimation of fastness and dynamics of bodies interacting with a free surface of a fluid.

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